QUASI-OPTIMAL SPACE-TIME FINITE ELEMENT METHODS

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We approach the problem of developing a stabilized finite element method by seeking an optimal interpolation of the exact solution. By using a methodology similar to that in [1], we examine the problem of finding an interpolation that is optimal (has least error) in a user-defined norm on a user-defined mesh, regardless of the energy norm associated with the original equations. The formulation is essentially that of a constrained minimization of the error in the chosen norm, the constraint being the original problem. Here, we are interested in initial and boundary value problems.

Our approach results in a Petrov-Galerkin formulation, in which the optimal weighting functions are determined by solving an auxiliary problem in the space-time domain. The operator in the auxiliary problem is the adjoint of that in the original problem, with a different forcing term. In implementation, this auxiliary problem is solved approximately. We investigate various approximations of the weighting functions in the context of initial and boundary value problems. For some simple approximations, we recover some well known time integrators for initial value problems.

We show that the quasi-optimal Petrov-Galerkin formulation is equivalent to a new class of stabilization methods. We compare the form of the quasi-optimal stabilization with other existing methods. In certain applications and with certain basis functions, the quasi-optimal method reduces to variational multiscale, residual free bubbles, or the SUPG method.

As examples, we consider a class of scalar time-dependent advection diffusion equations. In these examples, the trial functions have global support in space and local support in time; in particular, a piecewise linear basis in time is adopted. By considering individual space-time slabs, we formulate a time-marching scheme for the optimal projection. We present analytical and numerical studies of the properties of our time-marching scheme applied to the heat equation, advection diffusion equation, and other examples.

References

[1] P. E. Barbone, and I. Harari, "Nearly H1-optimal finite element methods", *Comput. Methods Appl. Mech. Engrg.*, 190, 5679-5690, 2001.